Mr. Michael T. Davis Algebra II Delta & Eta	Exponential Functions Unit III (2nd half) Test PRACTICE April 17, 2015

Name: Mr. Davis Solutions

1. WITHOUT a calculator, find 15% of 90. You must show work to receive full credit.

10% of 90 is 9   5% of 90 is $\frac{1}{2}$ of 10% or $\frac{1}{2}$ of 9

Therefore, 15% = 10% + 5% = 9 + 4.5 = 13.5

2. WITHOUT a calculator, find 1% of 144. You must show work to receive full credit.

$$1\%(144) = \frac{1}{100}(144) = \frac{144}{100} = 1.44$$

3. WITHOUT a calculator, find 300% of 45. You must show work to receive full credit.

$$300\%(45) = \frac{300}{100}(45) = 3(45) = 135$$

4. WITHOUT a calculator, find 37% of 60. You must show work to receive full credit.

37% = 0.37

$$\begin{array}{r} 60 \\ \times .37 \\ \hline 420 \\ 1800 \\ \hline \end{array} \rightarrow 22.20$$

22.20

5. WITHOUT a calculator, find 120% of 50. You must show work to receive full credit.

120% of 50 = 50% of 120 = $\frac{1}{2}$ of 120 = 60

Determine the answers to these questions WITH a calculator. Show work so you get full credit.

6. The rabbit population on Bunny Island is 8,540 today. If the population over the next year increases by 42%, then what will be the size of the population in one year? You must show work to receive full credit.

$$8540(1+42\%) = 8540(1+0.42) = 8540(1.42) = 12{,}126.8$$
$$\approx 12{,}127$$

7. The popular town of Smithville in 2003 was estimated to be 35,000 people with an annual rate of increase of about 2.4%. Write an exponential function to model future growth in Smithville. Use the function to determine the approximate population size in 2012. You must show work to receive full credit.

$$f(x) = 35{,}000(1+2.4\%)^x = 35{,}000(1.024)^x$$

$$f(9) = 35{,}000(1.024)^9 \approx 43{,}327.9 \approx 43{,}328$$

8. Marisa invests \$300 in a savings account with an annual interest rate of 5%. Write an exponential function to model the future value of the investment. Use the function to determine the value of the investment in 15 years. You must show work to receive full credit.

$f(x) = 300(1+5\%)^x = 300(1+.05)^x = 300(1.05)^x$

$f(15) = 300(1.05)^{15} \approx \$623.68$

9. Matt bought a new car at a cost of \$25,000. The care depreciates approximately 15% of its value per year. Write an exponential function to model the future value of the car. Use the function to determine the approximate value of the car in 9 years. You must show work to receive full credit.

$f(x) = 25{,}000(1-15\%)^x = 25{,}000(1-0.15)^x$

$f(x) = 25{,}000(.85)^x$ $\quad f(9) = 25000(0.85)^9 \approx 5{,}790.42$

10. A super ball is dropped from a height of 10 feet. On its first rebound, the ball rises up 8 feet, and on its second rebound, the ball rebounds (rises up) 6.4 feet. Write a function to model the height of the ball per each rebound. Determine on which rebound the ball reaches a height of about 2 feet. You must show work to receive full credit.

$b = 0.8$ $\quad f(x) = 10(0.8)^x$ $\quad f(3) = 10(.8)^3 \approx 5.12$ $\quad f(4) = 10(.8)^4 \approx 4.096$

$f(5) = 10(0.8)^5 \approx 3.28$ $\quad f(6) = 10(0.8)^6 \approx 2.62$ $\quad f(7) = 10(0.8)^7 \approx 2.097$

$f(8) = 10(0.8)^8 \approx 1.68$ $\quad$ 2 feet is reached at about the 7th bounce

11. Radium-226, a common isotope of radium, has a half-life of 1620 years. Professor Korbel has a 120 gram sample of radium-226 in his laboratory. Write an exponential function to model this decay. Determine how many grams of this 120-gram sample will remain after 8,100 years. You must show work to receive full credit.

$f(x) = 120\left(\frac{1}{2}\right)^x$ $\quad \frac{8100}{1620} = 5$ There are 5 1,620 year periods in 8,100 years.

$f(5) = 120\left(\frac{1}{2}\right)^5 \approx 3.75$ grams

12. A certain strain of bacteria that is growing on your kitchen counter doubles every 5 minutes. Assuming that you start with only one bacterium, determine the number of bacteria that will likely be present at the end of an hour and 35 minutes. You must show work to receive full credit.

$f(x) = 1(2)^x$ $\quad$ 1 hour & 35 min = 95 min

$f(19) = 1(2)^{19}$ $\quad \frac{95}{5} = 19$

$= 524{,}288$ bacteria

There are 19 five-minute periods in this time

13. Your fish tank contained 60 gallons when it was completely full. Since you filled the tank, water has evaporated so that there are now only 38.5 gallons of water. By what % did the water volume decrease? You must show work to receive full credit.

$$\frac{\text{change}}{\text{original}} = \frac{60-38.5}{60} = \frac{21.5}{60} = 0.358\overline{3} = 35.8\overline{3}\ \%$$

14. An exponential function contains the two points $(2,18)$ & $(3,54)$. Determine an equation for this function. Determine the y-value when $x = 5$.

x	0	1	2	3
y	2	6	18	54

$18 \cdot 3 = 54$ therefore $b = 3$ $f(x) = a(3)^x$

Divide 18 by 3 to get 6, etc.

we discover that $a = 2$

$f(x) = 2(3)^x$ $f(5) = 2(3)^5 = 486$

15. An exponential function contains the two points $\left(0, \frac{1}{2}\right)$ & $(3,32)$. Determine an equation for this function. Determine the y-value when $x = 6$.

$a = \frac{1}{2}$ because of $(0, \frac{1}{2})$ $f(x) = \frac{1}{2}(b)^x$

x	0	1	2	3
y	$\frac{1}{2}$	2	8	32

multiply $\frac{1}{2} \cdot 4$ to get 2, etc.. $b = 4$

$f(x) = \frac{1}{2}(4)^x$ $f(6) = \frac{1}{2}(4)^6 = 2048$

16. An exponential function contains the two points $(1,3)$ & $\left(4, \frac{1}{9}\right)$. Determine an equation for this function. Determine the y-value when $x = -1$.

Multiply 3 by $\frac{1}{3}$ etc...

x	0	1	2	3	4
y	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

$b = \frac{1}{3}$ Divide 3 by $\frac{1}{3}$ to get 9

$a = 9$ $f(x) = 9\left(\frac{1}{3}\right)^x$

$f(-1) = 9\left(\frac{1}{3}\right)^{-1} = 27$

17. Circle all the functions that represent exponential decay:

a. $f(x) = \frac{1}{2}(1.1)^x$ b. $f(x) = 7\left(\frac{8}{9}\right)^x$ c. $f(x) = 18\left(\frac{5}{4}\right)^{-x}$ d. $f(x) = 0.8(4)^x$

$0 < \frac{8}{9} < 1$ $0 < \frac{4}{5} < 1$

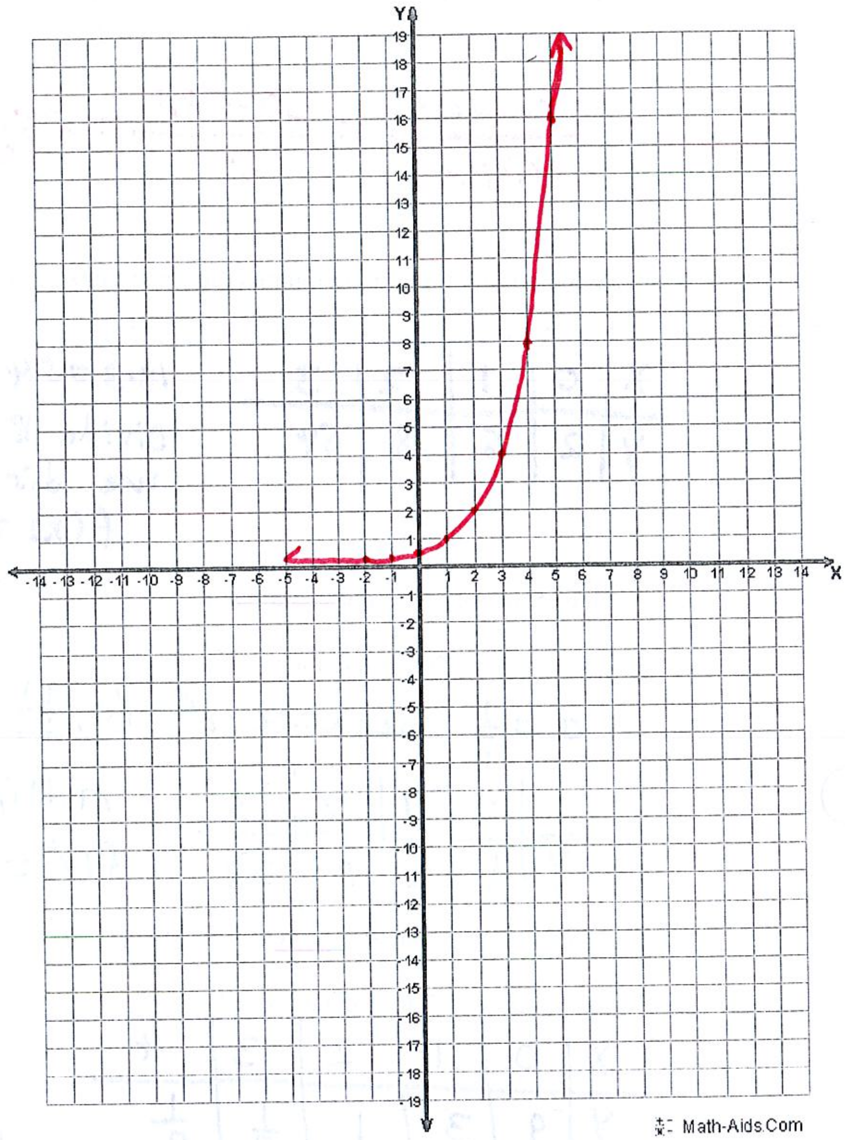
18. Circle all the functions that represent exponential growth:

a. $f(x) = \frac{6}{7}(0.95)^x$ b. $f(x) = 0.02\left(\frac{6}{11}\right)^x$ c. $f(x) = 3(1.5)^x$ d. $f(x) = 100\left(\frac{6}{7}\right)^{-x}$

$1.5 > 1$ $\frac{7}{6} > 1$

19. Given the exponential function $f(x)=\frac{1}{2}(2)^x$, fill in the x-y table, plot the points carefully and draw the graph neatly.

x	y
-2	$\frac{1}{8}$
-1	$\frac{1}{4}$
0	$\frac{1}{2}$
1	1
2	2
3	4
4	8
5	16



20. Given the function in # ~~10~~ 19 above, will the value of y or $f(x)$ reach zero? If so, what is the value of x for which $y=0$? If not, explain why the value of y will not reach $y=0$. Explain your answer clearly. f(x) will not reach zero. There is no exponent that turns 2^x into zero.

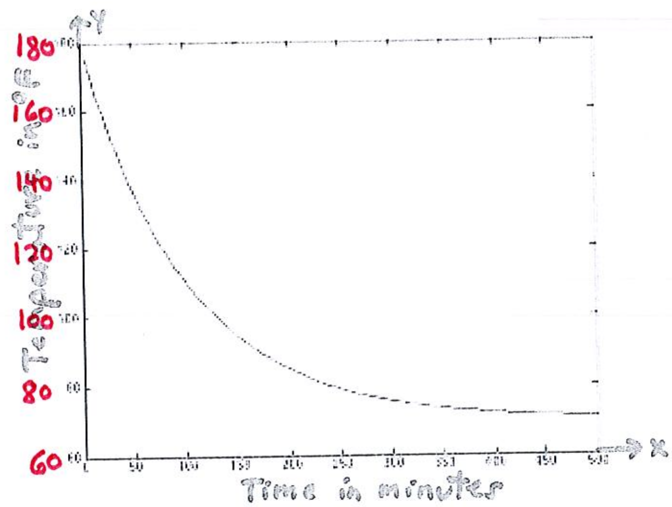
21. Suppose $f(x)=10\left(\frac{1}{3}\right)^x$ is a parent function. Write a clear and concise description of the transformation from $f(x)=10\left(\frac{1}{3}\right)^x$ to $g(x)=10\left(\frac{1}{3}\right)^{x-4}$

The graph of f(x) is shifted 4 units to the right

22. Suppose $f(x)=\frac{1}{3}(4)^x$ is a parent function. Write a clear and concise description of the transformation from $f(x)=\frac{1}{3}(4)^x$ to $k(x)=\frac{1}{3}(4)^x-3$

The graph of f(x) is shifted 3 units down

For #;s 23-26 .The graph below gives the temperature of a liquid as a function of time. The liquid is initially hot and cools toward the ambient temperature as time passes. The unit of the vertical axis is temperature in degrees Fahrenheit. The unit of the horizontal axis is time in minutes.



23. What was the initial temperature of the liquid?

180 °F

24. As time passes indefinitely, what temperature does the liquid seem to reach?

72 °F

25. Will the temperature of the liquid drop below the temperature you stated in #24 above? If not, then why not? If so, then why?

No, the liquid will only drop as low as the ambient or room temperature.

26. After how many minutes has the liquid cooled to half of its original temperature

about 160 minutes